PROBLEM SET 5

Considering time pressure, this problem set is for practice only --- not required or recorded.

It's OK to co-operate with classmates on problem sets. If you get stuck on a problem, don't waste a lot of time on it --- you have better things to do.

The following problems from Starr's *General Equilibrium Theory*, *2nd edition*, are assigned.

20.10

20.11

In addition, two problems adapted from the June 2008 qual are assigned, attached below.

Question 3

Consider an Arrow-Debreu economy with a full set of futures markets over time without uncertainty. Denote the present as date 0, and suppose there are a finite number of future periods, T.

Parts (i) and (ii) ask how saving and investment take place in this economy:

(i) Household h has a large endowment dated in periods T and T -1 but zero endowment dated 0, 1, 2, ..., T -2 . Household h wants relatively constant consumption throughout the periods 0, 1, 2, … , T. How can h arrange desired consumption using the futures markets?

(ii) Firm f has profitable investment opportunities at dates 0 and 1 that will produce marketable outputs at date k, k+1, … , T (where $2 < k < T$). How can f arrange to buy and pay for its 0 and 1 inputs using the futures markets?

(iii) Let there be n commodities available at each date t. The full commodity space is R^N where $N = n(T+1)$. Let equilibrium production in the economy be $y \in R^N$, $y = (y_0, y_1, \ldots, y_T)$ where the typical $y_t \in R^n$, represents period t output. y_t may differ from y_{t+1}. That is, equilibrium output may vary over time, despite the presence of a full set of futures markets and the absence of uncertainty. How can that happen?

Question 4

Consider the following group decision-making mechanism. The Arrow Social Choice conditions may be summarized as: transitivity, non-dictatorship, independence of irrelevant alternatives, Pareto principle, unrestricted domain. Which of the Arrow social choice conditions does the decision procedure below fail? Which does it fulfill? Explain fully.

Group Ranking Procedure: The choice set X consists of N (finite positive integer) alternatives, A, B,C, ... There are three voters. Each voter submits a ballot ranking the alternatives in order of preference. The voting procedure then gives each voter's top place choice a weight of N; the second place choice is given a weight of N-1; and so forth. For each alternative, the weighted votes of all the voters are then added up. The alternatives are then given a ranking in order of weighted vote total, highest total most preferred. A tie-breaking rule may be needed.

The question ends here.

You may find the following example useful, with four alternatives listed in rank order.

Profile 1		
Voter1	Voter 2	Voter 3
A	B	B
B	A	A
C	C	C
D	D	D

Profile 2		
Voter1	Voter 2	Voter 3
A	C	C
C	D	D
D	B	B
B	A	A

For reference, a restatement of the Arrow social choice theory is presented below.

A Summary of Arrow Social Choice Theory

The Arrow (Im) Possibility Theorem can be stated in the following way. We'll follow Sen's treatment.

X = Space of alternatives; X is assumed to have at least three distinct elements.

Π = Space of transitive strict orderings on X

H = Set of voters, numbered #H

$\Pi^{\#H}$ = #H - fold Cartesian product of Π , space of preference profiles

f: $\Pi^{\#H} \rightarrow \Pi$, f is an Arrow Social Choice Function.

P_i represents the preference ordering of typical household i. $\{P_i\}$ represents a preference profile, $\{P_i\} \in \Pi^{\#H}$. P represents the resulting group (social) ordering.
" x P_i y " is read "x is preferred to y by i" for $i \in H$

P (without subscript) denotes the social ordering, $f(P_1, P_2, ..., P_{\#H})$.

Unrestricted Domain: Π = all logically possible strict orderings on X.
$\Pi^{\#H}$ = all logically possible combinations of #H elements of Π.

Non-Dictatorship: There is no $j \in H$, so that x P y $\Leftrightarrow$ x P_j y, for all x, y $\in$ X, for all $\{P_i\} \in \Pi^{\#H}$.

(Weak) Pareto Principle: Let x P_i y for all $i \in H$. Then x P y.

For $S \subseteq X$, Define $C(S) = \{ x \mid x \in S, \ x P y, \text{ for all } y \in S, y \neq x \}$

Independence of Irrelevant Alternatives: Let $\{P_i\} \in \Pi^{\#H}$ and $\{P'_i\} \in \Pi^{\#H}$, so that for all x, y $\in S \subseteq X$, x P_i y if and only if ($\Leftrightarrow$) x P'_i y. Then C(S) = C '(S) .

General Possibility Theorem (Arrow): Let #H be finite, $\#X \geq 3$. Then there is no f: $\Pi^{\#H} \rightarrow \Pi$ satisfying (Weak) Pareto Principle, Independence of Irrelevant Alternatives, Unrestricted Domain, and Non-dictatorship.